Optimal Design of an Automotive Rear Axle Housing

Diane L. Peters
Kettering University
dpeters@kettering.edu

Yaomin Dong
Kettering University
ydong@kettering.edu

Vimal Patel
Kettering University
pate9936@kettering.edu

Abstract

This paper describes the optimization of the rear axle housing for a particular car, the 2013 Ford Mustang Shelby GT500. This optimization was conducted using the optimization and FEA capabilities of NXTM, the product development solution from Siemens PLM Software, with the objective of minimizing weight subject to constraints on the maximum stress in the housing. The optimization was conducted twice, once with a single design variable and once with five design variables. In both cases, the optimization problem converged to an optimal design, and the weight was reduced. In the first case, the weight of the housing was reduced by 15%; in the second case, it was reduced by 47%. These reductions could be expected to have a large impact on fuel economy and performance of the vehicle.

Introduction

As fuel economy requirements become more demanding, decreasing the weight of vehicles is critical. However, at the same time, it is necessary to meet performance requirements and manufacturability constraints. Some components' configuration may be the result of past decisions that were appropriate at the time; as other aspects of the vehicle change, technology improves, and new analysis techniques are developed, these legacy parts are good candidates for re-design. The axles are critical to the vehicle, as they support its weight, and producing an optimal axle design is important. In this work, a specific axle is considered: the 8.8-inch rear axle center housing from a 2013 Ford Mustang Shelby GT500. This axle is optimized for minimum weight, subject to a set of constraints based on the performance requirements for the axle, with the optimization carried out within the NX software.

This paper is organized as follows: First, the background for this project is given, including a discussion of the general design of automotive axles as well as some previous design and optimization work regarding axles. Next, the specific problem formulation is given. In the section after that, the results of the optimization are given, followed by conclusions.

Background

Automotive axles are a critical part of vehicles. The axles may be fixed to the wheels and rotating with them (live axle) or fixed to the vehicle with the wheels rotating about the axle. In the live-axle suspension system, axles both transmit driving torque and maintain the position of the wheels relative to each other and to the vehicle body. The axles also bear the weight of the vehicle and cargo. Non-driven axles, in contrast, serve only as a suspension and steering component. Vehicles typically have axles in both the front and rear; in a rear-wheel drive vehicle, the rear axle assembly is a driven or live axle, while in front-wheel drive vehicles, the rear axle is a non-driven axle. In the case where a vehicle has a driven rear axle, a housing contains and supports other parts of the axle assembly and forms a reservoir for lubricant. This housing consists of a central housing with axle tubes and is attached to the vehicle body through a suspension that involves springs, shock absorbers, and control arms, as described and shown in Chris and James (2010).

There are a number of variations on the basic rear axle housing. The housing may have a removable carrier or an integral carrier and may be semi-floating, three-quarter floating, or full-floating. A typical semi-floating axle has a half shaft connected directly to the wheel hub, with the outer end supported by a bearing inside the axle casing (Kaven, 2015). In the case of the three-quarter floating axle, the wheel hub is supported by a single bearing in the center of the hub, and the wheel hub runs on an axle housing, with the axle shaft keyed to the hub (Kaven, 2015). Finally, the full-floating axle features a wheel hub supported by two bearings, running directly on the axle housing, with the axle shaft fastened to the wheel hub flange through a coupling (Kaven, 2015). In this work, the axle considered is a full-floating axle, similar to that described in "The Ford Explorer 8.8-Inch Rear Axle" (n.d.).

Optimization has been applied to many different aspects of vehicle design at different scales, including both entire vehicles (Kokkolaras et al., 2004) and components and subsystems of vehicles. These studies have included powertrains and power management (Filipi et al., 2004), active suspensions (Frühauf, Kasper, & Lückel, 1986; Fathy, Papalambros, Ulsoy, & Hrovat, 2003), passive suspension systems (Chatillon, Jezequel, Coutant, & Baggio, 2006), and axles (Yimin, Xiangdong, & Qiaoling, 2002; Bin, Qinghong, Rui & Yanping, 2005). In Bin et al. (2005), a mass reduction of 4.2 kg was achieved, corresponding to a weight reduction of 41.2 N.

An optimization of the axle could be done in several ways: as sizing, shape, or topology. In this case, the problem is formulated as a sizing optimization, since this reduces the complexity of the problem. In applying optimization to the automotive axle, a simplified model can be used with analytical equations, or a more complex, "black box" optimization can be performed using FEA software or other complex, higher fidelity models. In this work, the latter approach is used, as it was in Bin et al. (2005).

Problem Formulation

The optimization of the housing was carried out using the advanced simulation application of Siemens NX 11 with a NASTRAN solver. The rear axle housing was modeled, with the material specified as grey cast iron. The material properties, as listed in Table 1, were incorporated into NX and used in this work. The sequence of processes conducting the optimization process is given in Figure 1.

Table 1. Material properties used for grey cast iron (G60).

Property	Value
Young's Modulus	158 GPa
Poisson's Ratio	0.28
Bulk Modulus	83333 MPa
Shear Modulus	42969 MPa
Isotropic Relative Permeability	10000
Compressive Ultimate Strength	382 MPa
Tensile Ultimate Strength	632 MPa

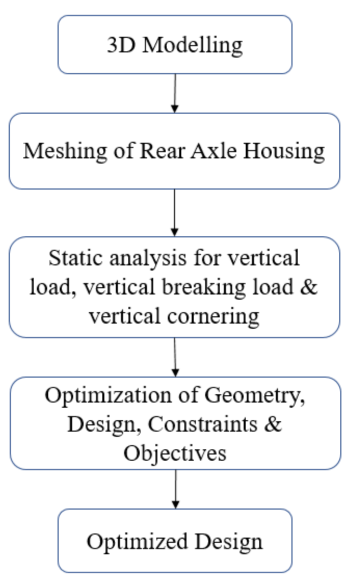

Figure 1. Process methodology for optimization (Patel, 2017). Reprinted with permission.

The 3D model of the housing, prior to optimization, is shown in Figure 2.

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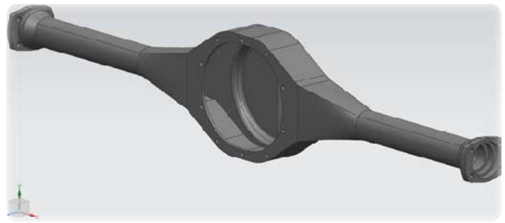

Figure 2. CAD model of semi-floating solid rear axle housing (Patel, 2017).
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The optimization was formulated to minimize weight, subject to constraints on deflection and stress, with the weight, deflection, and stress determined through the CAD software and FEA analysis. A single design variable was chosen, which impacts the entire design based on the geometric relationships within the axle housing. The control sketch for the optimization is shown in Figure 3.

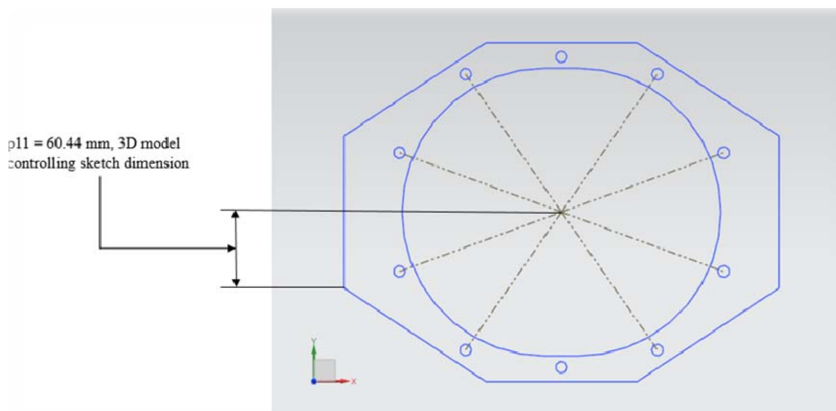

Figure 3. Control sketch for the rear axle housing (Patel, 2017).
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In the initial optimization, the sole design variable was the dimension *p*11; the problem, therefore, can be formulated in standard negative-null form as

$$
\begin{aligned}
&\min_{p11}\ AxleWeight \\
&p11 \in \{55 \text{ mm} \le p11 \le 65 \text{ mm}\} \\
&\text{subject to:} \\
&MaxStress - StressLimit \le 0 \\
&MaxDeflection - DeflectionLimit \le 0
\end{aligned}
\tag{1}
$$

with the upper and lower limits on *p*11 chosen due to manufacturing considerations. In a second optimization problem, the shell thickness and the chamfer over the center of the differential housing were also varied. No other changes were made to the optimization problem for the second optimization.

In the full range of vehicle operation, the maximum stresses that the housing should withstand include the following (Saxena, 2014):

- Torsional stress due to driving and braking torque
- Shear and bending stresses due to the weight of the vehicle
- Tensile and compressive stresses due to cornering forces

This optimization was carried out as a static optimization, with calculations performed to determine what static loads would correspond to the driving and braking torque, and to the effects of cornering.

In calculating the stress and deflection, certain assumptions must be made about the vehicle and its weight distribution. It was assumed that the curb weight of the vehicle is 3850lb, that the EPA test weight of 300lb is used, and that the weight distribution is such that the front axle carries 57% of the total weight and the rear axle carries 43% (Data, 2012). It is further assumed that the full vehicle weight load is applied at the spring seat locations shown in Figure 4.

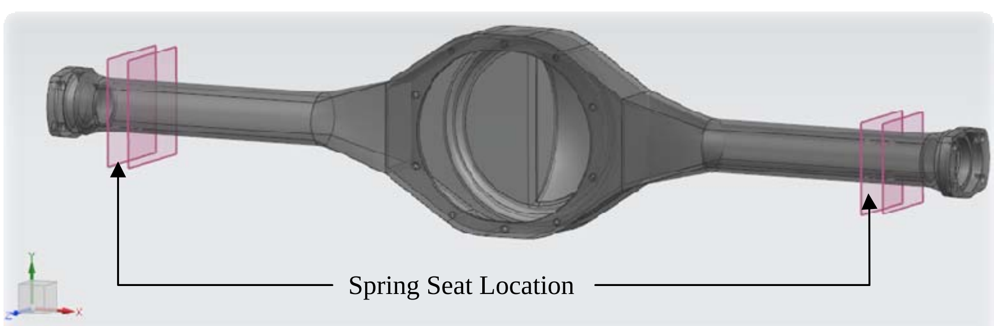

Figure 4. Model of 8.8-inch rear axle housing showing spring seat location (Patel, 2017). Reprinted with permission.

To determine the effects of driving and braking forces, several intermediate calculations are necessary. The effective radius of the wheel needs to be calculated, based on (Mazzei, 2012):

$$r_{eff} = 0.5\left(\left(25.4\frac{\text{mm}}{\text{in}}\right)(20\text{ in}) + 0.02(35)(285)\right) - 4\text{ mm} = 349.75\text{ mm} \tag{2}$$

The frictional force between the car and the road also needs to be calculated. To perform this calculation, it is assumed that under normal driving conditions, each tire supports half of the load that is applied to the rear axle. Assuming a coefficient of friction μ=0.85 between the road and the tire (Davis & Hoff, 2003), the frictional force on each tire will be given by

$$F_{fric} = \mu\left(\frac{0.43W}{2}\right) = 758.4 \text{ lb } \left(3373.6 \text{ N}\right) \tag{3}$$

This frictional force produces both a torque about the axle, based on the effective radius of the wheel, and a bending moment at the center of the outboard bearing. The torque is given by

$$\tau = F_{fric} r_{eff} = \left(3373.6 \text{ N}\right)\left(349.75 \text{ mm}\right) = 1179.9 \text{ N-m} \tag{4}$$

and the bending moment is given by

$$M = F_{fric} l \tag{5}$$

Where l is the distance between the center planes of the road wheel and the outboard bearing of the axle; for this particular vehicle, that dimension is 3.86in (Chevrolet and Ford, 2012-2013), or 98.044mm, resulting in a bending moment of 330.8N-m. These two loads are assumed to be present at all times.

During a sudden braking event, additional loads are present. In its top gear, the 2013 Ford Mustang GT500 produces a maximum torque of 855.5 N-m at 4000rpm (Data, 2012), and the driveline efficiency and clutch efficiency are both assumed to be 90% (Davis & Hoff, 2003). When the vehicle is traveling straight, it is assumed that it will be traveling at 60mph (26.82 m/s) and will have a stopping distance of 20m. If the acceleration is constant during this braking event, then the stopping time and acceleration can be calculated using basic kinematic equations.

$$t = \frac{2\left(x_f - x_0\right)}{v_f + v_0} = \frac{2\left(20 \text{ m}\right)}{0 \text{ m/s } + 26.82 \text{ m/s}} = 1.49 \text{ s} \tag{6}$$

$$a = \frac{v_f^2 - v_0^2}{2\left(x_f - x_0\right)} = \frac{-\left(26.82 \text{ m/s}\right)^2}{2\left(20 \text{ m}\right)} = -17.98 \text{ m/s}^2 \tag{7}$$

The force required to produce this acceleration can be calculated, and then the torque can be found, based on the perpendicular distance from the center of the differential housing to the end of the side flange.

$$F_{stop} = \left(809.44 \text{ kg}\right)\left(-17.98 \text{ m/s}^2\right) = -14560 \text{ N} \tag{8}$$

$$\tau_{stop} = \left(-14560 \text{ N}\right)\left(0.662 \text{ m}\right) = -9636 \text{ N-m} \tag{9}$$

This loading is also considered in the total loading of the axle housing for the optimization.

When the vehicle is cornering, it is assumed that the lateral acceleration for the cornering maneuver is 0.95g, and the vertical height of the center of gravity is 24.2 in. The track width at the rear end of the vehicle is 62.5 in Kaven (2015). The cornering force is given by Equation 10 (Data, 2012):

$$F_{corner} = 0.43Wg_c\left(0.5 + g_c\frac{h_{vt}}{b_r}\right) \tag{10}$$

Where W is the total weight, including the EPA standard test weight, as previously specified; 0.43 represents the portion of the weight on the rear axle; g_c is the lateral acceleration for cornering; h_{vt} is the vertical height of the center of gravity of the vehicle, measured from the ground (24.213in, or 615mm); and b_r is the track width at the rear end of the vehicle. The cornering force can then be calculated as 1472lb, or 6548N.

The problem was then solved using the NX NASTRAN FEA capabilities integrated into NX, using the standard NX optimizer. As indicated in Figure 3, a mesh had to be generated for the FEA solution. In this problem, a 3D tetrahedral mesh was used, with the element type CTETRA. A standard element size was used for the entire optimization; to ensure that the initial element size selected was sufficiently small, the recommended element size from the software was divided by two, resulting in a mesh element size of 7.25mm. The meshing is shown in Figure 5.

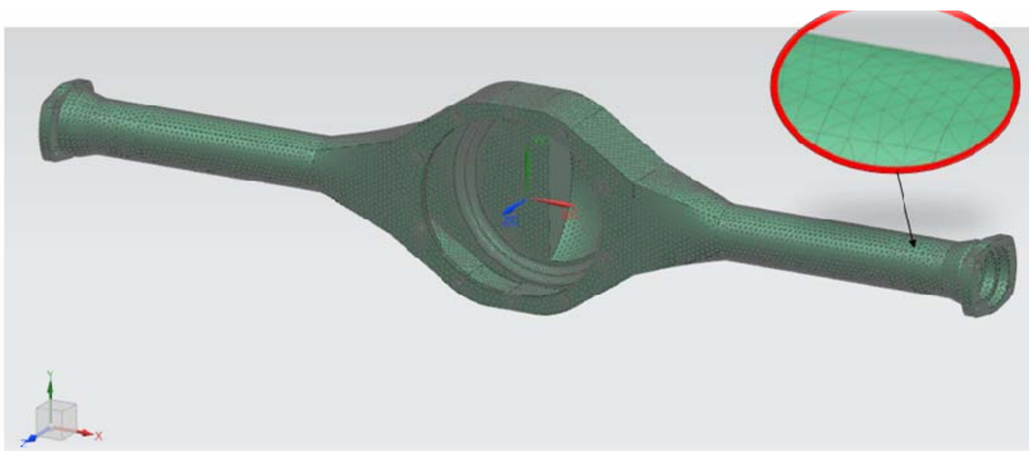

Figure 5. Visualization of 3D tetrahedral mesh size (Patel, 2017).
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Prior to performing the optimization, a standard structural analysis was performed to validate the finite element model (mesh type and size, constraints, loads, etc.), and to obtain the baseline results of structural responses (stresses, deformation, etc.). The results of this simulation are shown in Figures 6 and 7, with Figure 6 showing the displacement and Figure 7 showing the von Mises stress.

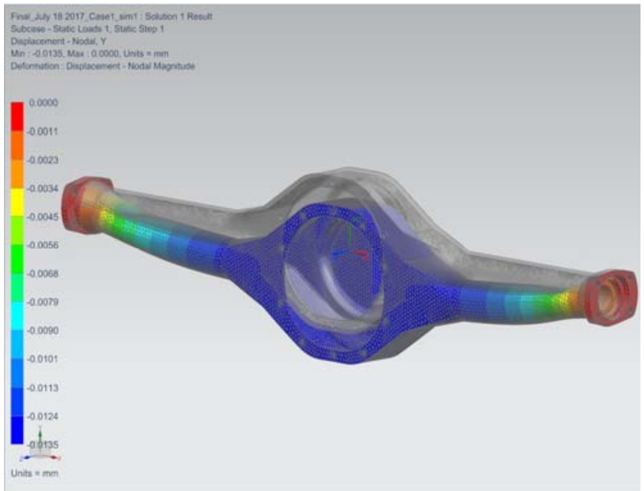

Figure 6. Deflection of original design (Patel, 2017). Reprinted with permission.

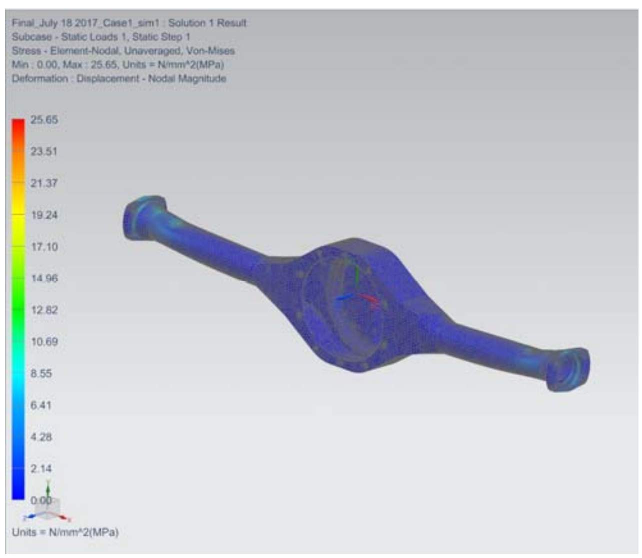

Figure 7. Stress profile of original design (Patel, 2017). Reprinted with permission.

The initial design was feasible, as the maximum stress of 25.65MPa was well below the yield stress of the material.

Results

The optimization is then performed using “Advanced Solution Process” based on the “baseline” analysis described above. Both optimization problems did converge to feasible solutions. In the first case, the problem converged quickly, after only three iterations; in the second optimization, the problem took seven iterations to converge.

First Optimization

When the first optimization was performed, with only *p*11 as the design variable, the optimization converged after 3 iterations, as shown in Figure 8, with *p*11 taking on its minimum value of 55 mm.

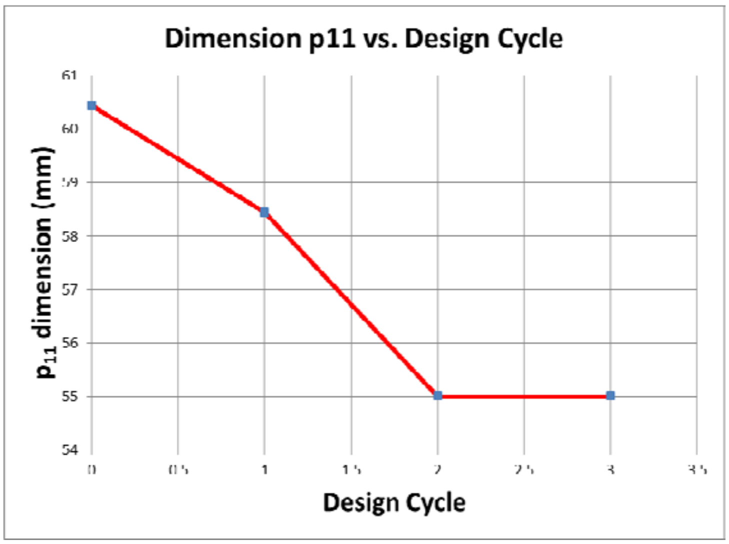

Figure 8. Convergence of first optimization (Patel, 2017). Reprinted with permission.

The initial weight of the rear axle housing was 187.8 N, with the final weight at 160.4 N, representing a 15% reduction. The new design is shown in Figure 9, with the decrease in weight with changes in *p*11 shown in Figure 10.

ISBN 978-1-60643-379-9

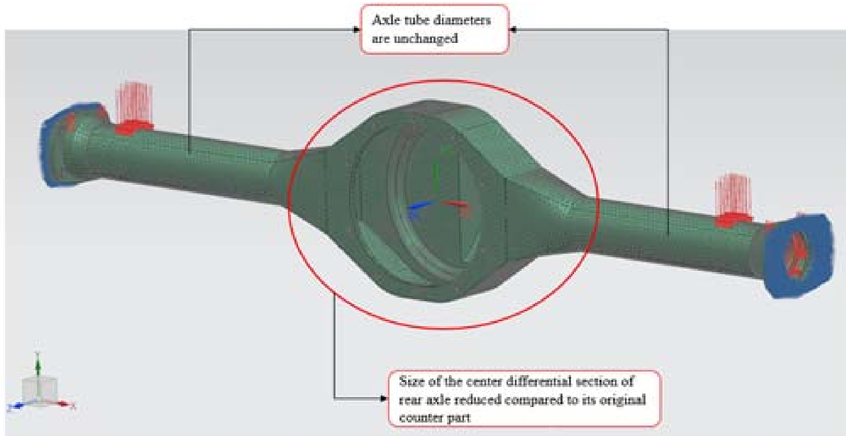

Figure 9. Design resulting from first optimization (Patel, 2017).
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Note that the optimization converged when the dimension *p*11 was at its lower bound and that none of the stress constraints or deflection constraints were active. This suggests that there is still an opportunity to improve the design further, if doing so would produce a manufacturable design.

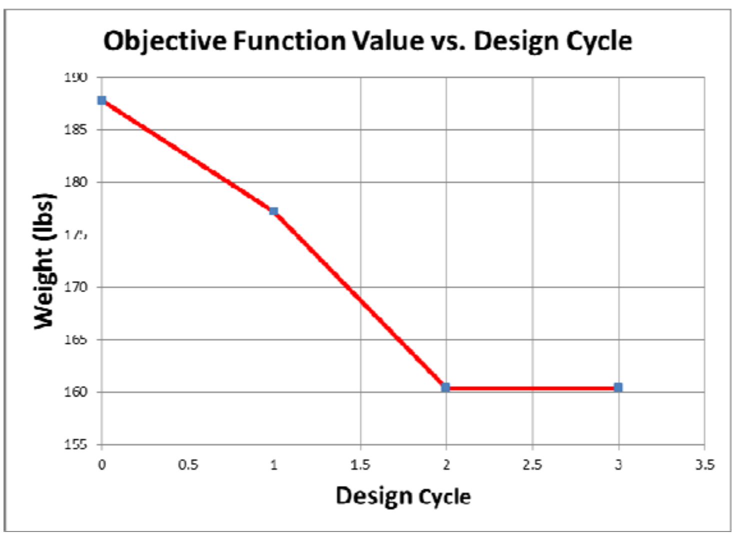

Figure 10. Decrease in weight for first optimization (Patel, 2017). Reprinted with permission.

Second Optimization

In the second optimization, there were five design variables, as shown in Figure 11. The optimization converged after seven iterations, with the final values given in Table 2 and the progress towards optimality shown in Figure 12. The maximum stress constraint is active for this solution.

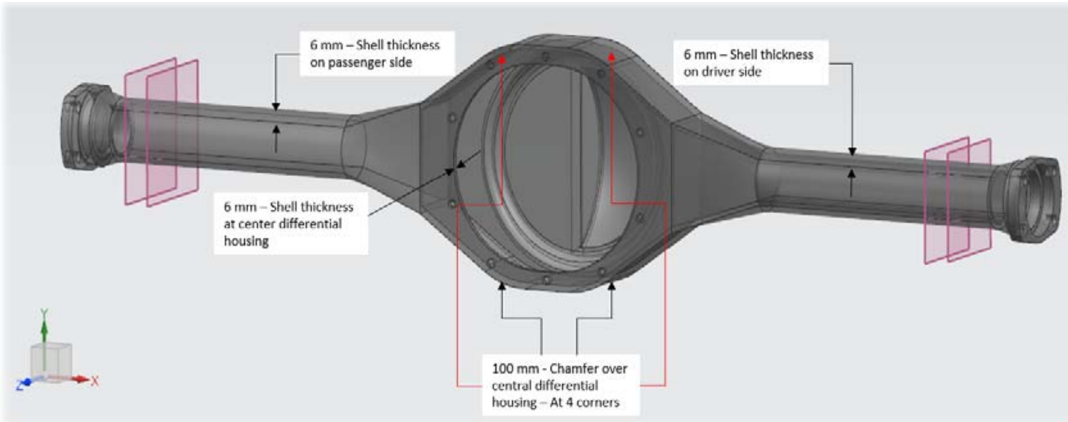

Figure 11. Design variables for second optimization

Table 2. Results of second optimization.

	Initial Value	**Final Value**
Design Objective Function Results		
Minimum Weight [N]	186.52	98.7
Design Variable Results		
Control sketch dimension, *p*11	60.44	55.03
Shell thickness of the center differential housing	6	3
Shell thickness of the right axle tube	6	3
Shell thickness of the left axle tube	6	3
Chamfer over the center differential housing	100	109.8

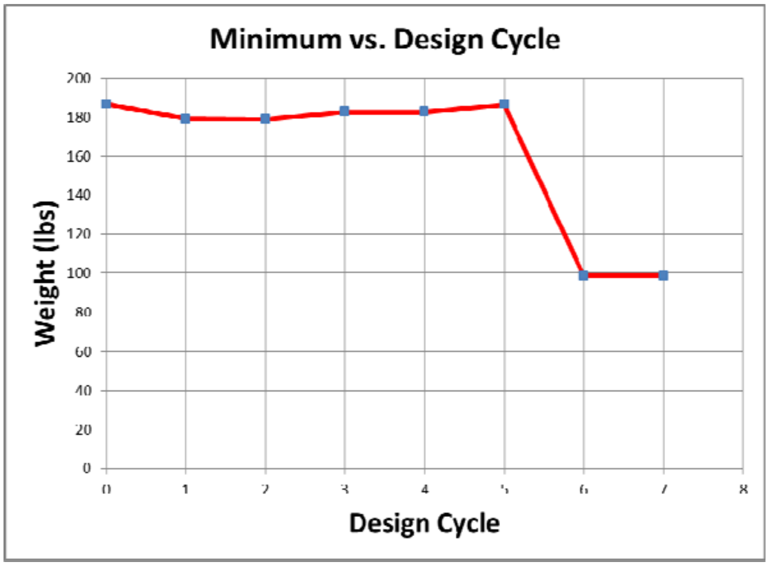

Figure 12. Convergence of second optimization (Patel, 2017). Reprinted with permission.

The new design of the axle is shown in Figure 13, below. There are noticeable differences between this design and that shown in Figure 9, from the first optimization. The dimension *p*11 has still taken on the value of its lower bound; however, the shell thicknesses have also been decreased to their lower bounds, and the chamfer on the housing has been increased slightly. The chamfer size is not at an upper or lower bound for the problem.

Note that the new weight of the housing is 98.7N, as opposed to the weight of 160.4 N found in the first optimization. This represents a reduction of 87.8N from the original weight, or 47%. This is a significant reduction, which can be expected to have a large impact on vehicle dynamics and fuel economy.

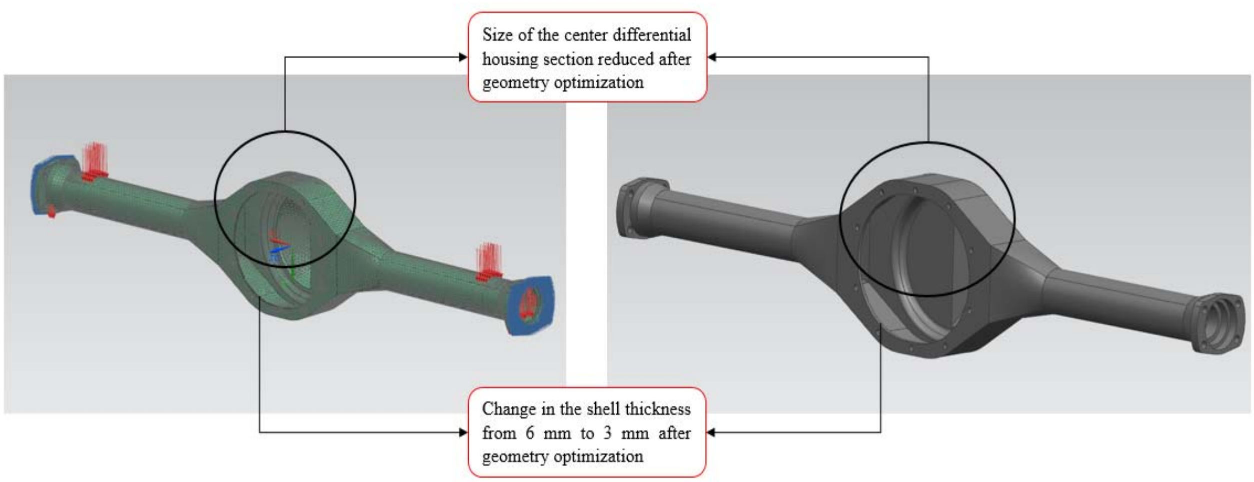

Figure 13. Optimization results for optimization #2 (Patel, 2017). Reprinted with permission.

Conclusion

In this paper, the optimization of a rear axle for a 2013 Ford Mustang Shelby GT500 was formulated and solved. This optimization showed that, by varying a single design variable, a 15% weight reduction could be achieved while still satisfying constraints on the axle. Increasing the number of design variables allowed for a greater weight reduction of 47% compared to the original weight. This weight reduction can be expected to have a substantial impact on fuel economy; simulations of the vehicle performance with the reduction in weight were carried out and are reported in Patel (2017).

Future work should, at a minimum, include the dynamic conditions of cornering and braking, as they have a substantial impact on the problem. The optimization could also include the use of different materials such as various steel alloys and composite materials and could include a non-uniform material composition. Additional work could be done by optimizing the axle along with other components, in a system-level problem formulation. Finally, the results of the optimization could be prototyped and physical tests conducted, to verify that the solution found actually performs as intended.

Acknowledgments

The authors would like to thank the Graduate School of Kettering University for providing the graduate assistantships during this work and Dr. Arnaldo Mazzei for serving as a committee member of the graduate student's research.

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Biographies

DIANE L. PETERS is currently an assistant professor of Mechanical Engineering at Kettering University. She is a licensed professional engineer in Illinois and Michigan, a member of ASME and SAE, a senior member of IEEE, and a fellow of SWE. Dr. Peters may be reached at dpeters@kettering.edu.

YAOMIN DONG is currently a professor of Mechanical Engineering at Kettering University. He received his PhD in Mechanical Engineering at the University of Kentucky in 1998. Dr. Dong has extensive R&D experience in the automotive industry and holds multiple patents. Dr. Dong's areas of expertise include metalforming processes, design with composite materials, computer graphics, computer-aided engineering, and finite element analysis. Dr. Dong may be reached at ydong@kettering.edu.

VIMAL PATEL received his master's degree at Kettering University in 2017. Contact Mr. Patel at pate9936@kettering.edu.